Production Planning and Allocation of Re-Entrance Flows with Stochastic Quality

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***Abstract*—Circular production promotes economic growth while conserving resources and energy. In this study, one aspect of production systems, the repair loop of returned products, is considered. The production system is composed of multiple flexible workstations that can process returns with different quality levels. This system aims to complete order production and deliver it on time by optimizing the use of the given resource levels. A sequential decision-making approach is employed to obtain rolling production decisions over a finite planning horizon. At each decision period, the manufacturer needs to simultaneously determine the optimal repair allocation plans for first returns and their potential second returns that will be back after certain periods. For traceability and quality reasons, a product at each return is required to be repaired on the same workstation; otherwise, a penalty cost is imposed. A stochastic mixed-integer programming model is developed based on sample average approximation (SAA) method, where the quality level of the second return is stochastic. Numerical results demonstrate that the value of the stochastic solution is significant compared with the average performance of solutions obtained by the existing deterministic model. Useful guidelines related to stochastic solutions and optimal capacity allocation patterns are also provided for production practices.**

***Index Terms*—Circular economy, repair loops, uncertain return quality, sequential production decision-making, multiple resources, stochastic mixed-integer programming.**

I. Introduction

IN the conventional linear economy of production and consumption, raw materials are processed into finished final products, which are then sold to customers, used by customers, and disposed of. The idea of the "circular economy" has gained significant traction in the political, business, and public spheres in recent years. By extending the service life of products through, for example, reuse of products in a suitable state, remanufacturing of returned products, and recycling of raw materials, the circular economy seeks to reduce wastes and the consumption of finite raw materials [1].

One example for circular production systems comes from automotive products like car engine. Returns with varying quality levels have various characteristics and valuations in the market. "As good as new" repaired engines whose condition is fully functional will be sold to the primary market to produce new vehicles. Repaired engines with lower quality level will be used for original equipment service/independent aftermarket [2]. This implies that the values of resources differentiate based on which quality level of returned product it is used to process. In this context, industrial practitioners struggle with identifying the quality of returns, differentiating and optimizing the use of available resources to efficiently satisfy customer demands with minimization of operational costs. In addition, product traceability plays an important role in quality management. Motivated by these prospects, in this work we study the circular production decision problem on first and their potential second returns simultaneously.

This work investigates short repair loop of circular production with re-entrant flows. Repair is described as 'bringing back to working order', 'making it as good as new', 'recreating its original function after minor defects', 'replacing broken parts' [1], [3], [4]. In the context of production system explored here, production decisions include which workstation and when first and second repairs for returns should be processed. The objective is to fulfill the customers' demand for repaired products as cost-effectively as possible. Compared to traditional manufacturing systems, production decisions are more complicated in circular production systems with re-entrant flows. This challenge raises in part from the remanufacturer's limited control over the input flows of returned products from customers both in terms of quantity and quality. In addition, to achieve product processing information traceability, the input product flow of the system is complexly coupled. The repair plans for products of first returns and their corresponding second returns that will be back after a period of time are optimized at the same workstation at the same time.

The academia is paying increasing attention to production decision problem in circular production systems. In general, the optimal production quantities for each period are deter-

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mined with the goal of either minimizing total costs including production, inventory, disposal costs, and so on [3], [5], or maximizing profit by optimizing the difference between sales revenue and total costs [2], [8]. Ndhaief et al. [5] study a production system consisting of one manufacturing unit, one remanufacturing unit and a subcontractor. Literature [3], [6], and [7] investigate a production planning/scheduling problem for a remanufacturing system comprising multiple stages, i.e., disassembly, refurbishing/reprocessing, and reassembly. Several practical stochastic factors, such as new and reman product demand, uncertain quality of core returns, remanufacturing cost, resource consumption, and so on, are addressed in [3], [8], and [9]. Environmental issues, such as optimization of carbon emissions, have also been discussed by Ndhaief et al. [5] and Chang et al. [10] in manufacturing and remanufacturing system. In the above mentioned works, some models in [3], [6], and [7] optimize the production plan for a single return cycle. Zhang et al. [6] and Jin et al. [7] optimize a single return cycle across multiple resources. New product and a single re-entrance flow are optimized on a single resource by Ndhaief et al. [5] and Chang et al. [10]. This work addresses the optimization of the multiple return loops across multiple resources while retaining the records of the resource processing history. We adapt the model for appointment scheduling proposed in [11] for the application in circular production systems with re-entrant flows. Indeed, the circularity feature is in common with many other applications such as healthcare, semi-conductor.

The main contributions of this study are summarized as follows. First, we define the optimization problem of production planning and allocation in repair loop production systems over a multi-period horizon. This issue is meaningful and deserves to be investigated to promote sustainable manufacturing development. Second, through employing SAA and a multi-period sequential decision-making approach, we model and address the uncertainties about the return quality such that different repair processes are considered. The subscripts of workstation are introduced to formulate soft constraints to ensure that the first and second repairs of one returned product are assigned to the same workstation as much as possible to track processing information. Third, the value of the stochastic solution is illustrated compared with existing production decision models. Practical implications are reveled to better apply the proposed model and solution in real-world production systems.

II. Problem Description

We consider a make-to-order circular repair production systems with re-entrant flows. The system is composed of multiple parallel flexible production workstations. Each workstation has a given capacity level to repair different quality returns to satisfy customer demands. A single operation for repairing returns with certain quality level is carried out in a single workstation at any given time, or the workstation is idle. We assume the return has three quality levels. The quality order of levels 1 to 3 is: $Q_1 > Q_2 > Q_3$. Returns with varying quality levels require their own specific repair operations. Variability in unit capacity consumption, fixed setup and unit capacity overtime and idle cost has been explicitly considered to imply the differences in repair process of different quality returns. The higher the quality level of the return, the lower unit capacity usage, fixed setup, and unit capacity overtime and idle costs, and vice versa. The order demand (*i.e.*, Q_1 returns with quality level one) received at each period needs to be allocated to a certain workstation on a certain period before the due date for the first repair. After first repair, the Q_1 returns are processed into "as good as new" products and are delivered to the customer. After a certain period when first-repair finished products enter the market, some of them are returned to the manufacturer as second return for second repair. Due to the impact of some factors, such as customer usage habits and usage duration, the associated quality of second return is uncertain to be quality level Q_2 or Q_3. Returns with lower quality levels are returned for longer interval, and vice versa. These returned products requiring a second repair need to be assigned to a workstation on a certain period for processing. After second repair, the Q_2 or Q_3 returns are processed into finished products with higher quality. The returns flow is shown as Fig. 1. To track processing information, the first and second repair process for the same one return need to be implemented on the same workstation. Our goal is to create a production plan for this system over a multi-period horizon with minimization of total cost including setup cost, capacity overtime and idle cost, and penalty cost related to processing information traceability.

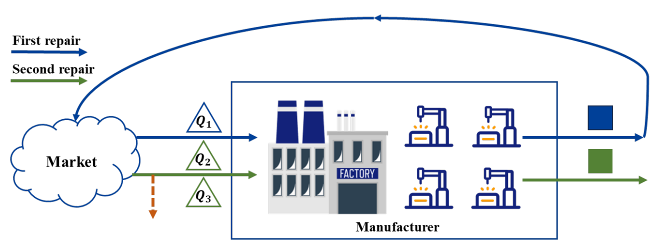

Fig. 1. The returns flow in the problem.

III. Multi-period Stochastic Mixed Integer Programming Method

A. Multi-period Sequential Decision-making Approach

A manufacturer needs to make production plans for the order received during R periods, indexed by $r \in \mathcal{R} = \{1, \ldots, R\}$. The planning window includes H periods, indexed by $h \in \mathcal{H} = \{1, \ldots, H\}$. Therefore, each period in the planning horizon at decision period r can be indexed by $t \in \mathcal{T} = \{r+1, \ldots, r+H\}$. We use a multi-period sequential decision-making framework to obtain the production plan. At the beginning of period r, we can observe the information on the number of process capacities for different quality levels of returns of each workstation already assigned to the returned products for first and second repairs on period $t \in \mathcal{T} = \{r+1, \ldots, r+H\}$. Then, the available process capacities can be updated. During period r, the manufacturer receives the order of returned products for first repair with their potential returns for second repair. At the end of period r, the optimization model is solved for a planning horizon of H days to allocate the first returns of the order received at period r and potential second returns to a specific workstation j on a certain period. After that, the plan created for period $r+1$ is put into action. Prior to solving the model on period $r+1$ for the following H periods, the newly arising order is gathered and the available capacities are updated. On period $r+2$, the plan already generated is executed. After capacity and order have been updated appropriately, the model is solved again for the next H periods. For the remaining stages of the R decision horizons, this pattern is replicated.

B. SAA-based Stochastic Programming Model Formulation

A manufacturer repairs one type of returned product with different quality levels through its J flexible workstations, indexed by $j \in \mathcal{J} = \{1, \ldots, J\}$. Each workstation $j \in \mathcal{J} = \{1, \ldots, J\}$ has a fixed setup cost $c_u^i, \ \ i = 1,\ 2,\ 3$ for repairing $Q_i, \ \ i = 1,\ 2,\ 3$ returns and can perform repair operations for $Q_i, \ \ i = 1,\ 2,\ 3$ returns at period $t \in \mathcal{T} = \{r+1, \ldots, r+H\}$ denoted by α_{jt}, β_{jt}, and γ_{jt}. α_{jt}, β_{jt}, and γ_{jt} is a capability parameter that equals one if workstation j is set up to repair a returned product with quality level $Q_i, \ \ i = 1,\ 2,\ 3$ at period t, and zero otherwise. Each workstation j has a regular production capacity c_{jt} at period t, and allocates $c_{jt}^i, \ \ i = 1,\ 2,\ 3$ available capacity for repairing $Q_i, \ \ i = 1,\ 2,\ 3$ returns at period t, respectively. Each $Q_i, \ \ i = 1,\ 2,\ 3$ return requires $d_j^i, \ \ i = 1,\ 2,\ 3$ units of processing capacity for repairing operation at workstation j, respectively.

At the beginning of a decision period r, the manufacturer receives a customized returned product order from the customer for the first repair. A set of $\mathcal{I}_1$ returns with a given committed delivery date M_1 of the order will return for second repair $M_{i_1}^2$ periods after the customer receives the first-repair finished products. The remaining set of $\mathcal{I}_2$ returns with a given committed delivery date M_2 will not return to the manufacturer again. The quality level of returned product $i_1 \in \mathcal{I}_1$ for second repair is uncertain to be Q_2 or Q_3 such that the repair process required by the return is uncertain. We use stochastic programming approach to model the decision-making problem under uncertainty. The random vector of the quality of second return is denoted by ξ. It is assumed that ξ is drawn from the distribution F. The solution methods for this problem typically depend on either assuming a prior distribution for F or utilizing a set of independent and identically distributed (i.i.d.) observations. We use the latter case, a set of i.i.d. observations of the random vector ξ, denoted by $\mathcal{S}\colon =\{\xi_i\}_{i=1}^S$, to formulate the stochastic mixed-integer programming based on SAA [12]. We assume that the observed data follows a Bernoulli distribution. Let $k_{i_1}^s$ denote $i_1 th$ component of the random vector in scenario s, where $k_{i_1}^s = 1$ if the quality level of second return i_1 is Q_2, and $k_{i_1}^s = 0$ if it is Q_3. The amount of time $M_{i_1}^2$ for a customer to return a Q_2 (Q_3) product for second repair is between $\underline{a}^f (\underline{a}^p)$ and $\overline{a}^f (\overline{a}^p)$. To promptly respond to customer demands, in this make-to-order system, we assume that there is no inventory of the returned product. The manufacturer arranges repair processing once the returned product is received. In addition, the system has no finished product inventory. Once the product has completed the repair process, it will be delivered to the customer.

At the end of a decision period r, the following decisions need to be made by the manufacturer: (1) assign Q_1 returns in set $\mathcal{I}_1$ and $\mathcal{I}_2$ to workstation j on period t for first repair, denoted by binary variable $x_{i_1jt}^1$ and $x_{i_2jt}^2$, respectively; (2) assign return i_1 with quality level Q_2 and Q_3 to workstation j for second repair on period t in scenario s, denoted by binary variable $y_{i_1jt}^s$ and $z_{i_1jt}^s$, respectively; Then, the indirect decisions are made: (3) workstation j's overtime processing capacity required to complete the assigned repair tasks of $Q_i, \ i = 1,\ 2,\ 3$ returns on period t in scenario s, denoted by continuous variables $o_{jt}^{is}, \ i = 1,\ 2,\ 3$; (4) workstation j's idle processing capacity after finishing its repair tasks of $Q_i, \ i = 1,\ 2,\ 3$ returns on period t in scenario s, denoted by continuous variables $v_{jt}^{is}, \ i = 1,\ 2,\ 3$.

We divide the above decisions that need to be made into two stages, similar in [13]. Among them, $x_{i_1jt}^1$ and $x_{i_2jt}^2$ are the first stage decisions to plan first repair which do not depend on stochastic parameters, *i.e.*, the quality of second returns. Variables with subscript s, $y_{i_1jt}^s$, $z_{i_1jt}^s$, o_{jt}^{is}, and v_{jt}^{is}, are the second stage decisions which depend on stochastic parameters. The second repair plan, capacity overtime and idle are determined in second stage. The first stage decision will affect the decisions in the second stage. The objective function is to minimize the total cost consisting of repair setup cost for processing $Q_i, \ i = 1,\ 2,\ 3$ returns, capacity overtime and idle cost, and penalty cost. Let $c_b^i, \ i = 1,\ 2,\ 3$ denote a unit overtime or idle cost of workstation performing repair operations for $Q_i, \ i = 1,\ 2,\ 3$ returns, respectively. To ensure that product processing information is traceable, a penalty is imposed if the second repair of the Q_2 or Q_3 return cannot be performed by the same workstation as in their first repair, with a unit penalty cost c_p^f and c_p^p.

Then, we establish a stochastic mixed-integer programming based on SAA. The constraints (1)-(21) and objective function (25) of the mathematical model are formulated in the following.

(a) Product assignment constraints: The below constraints ensure that each returned product for first repair and second repair will be assigned to one workstation on one period.

$$\sum_{j \in \mathcal{J}} \sum_{t=r+1}^{r+H} x_{i_1jt}^1 = 1, \forall i_1 \in \mathcal{I}_1, \tag{1}$$

$$\sum_{j \in \mathcal{J}} \sum_{t=r+1}^{r+H} x_{i_2jt}^2 = 1, \forall i_2 \in \mathcal{I}_2, \tag{2}$$

$$\sum_{j \in \mathcal{J}} \sum_{t=r+1}^{r+H} y_{i_1jt}^s = k_{i_1}^s, \forall i_1 \in \mathcal{I}_1,, s \in \mathcal{S}, \tag{3}$$

$$\sum_{j \in \mathcal{J}} \sum_{t=r+1}^{r+H} z_{i_1jt}^s = 1 - k_{i_1}^s, \forall i_1 \in \mathcal{I}_1, s \in \mathcal{S}. \tag{4}$$

(b) Processing setup constraints: The assignment variable $x^1_{i_1jt}$, $x^2_{i_2jt}$, $y^s_{i_1jt}$, and $z^s_{i_1jt}$ can only be possibly 1 if and only if workstation j is setup to be capable of performing repair operations for $Q_i,\ i=1,\ 2,\ 3$ returns at period t. This relationship can be expressed as follows:

$$x^1_{i_1jt} \le \alpha_{jt}, \forall i_1 \in \mathcal{I}_1, j \in \mathcal{J}, t \in \mathcal{T}, \tag{5}$$

$$x^2_{i_2jt} \le \alpha_{jt}, \forall i_2 \in \mathcal{I}_2, j \in \mathcal{J}, t \in \mathcal{T}, \tag{6}$$

$$y^s_{i_1jt} \le \beta_{jt}, \forall i_1 \in \mathcal{I}_1, j \in \mathcal{J}, t \in \mathcal{T}, s \in \mathcal{S}, \tag{7}$$

$$z^s_{i_1jt} \le \gamma_{jt}, \forall i_1 \in \mathcal{I}_1, j \in \mathcal{J}, t \in \mathcal{T}, s \in \mathcal{S}. \tag{8}$$

(c) Returned interval requirements: After $M^2_{i_1}$ period when the first-repair finished product enters the market, it returns the manufacturer again with Q_2 or Q_3 quality for second repair. $M^2_{i_1} \in [\underline{a}^f,\ \overline{a}^f]$ ($M^2_{i_1} \in [\underline{a}^p,\ \overline{a}^p]$) if the quality level of second return is Q_2 (Q_3). The following constraints ensure the second return to be back and allocated in the anticipated time window for second repair.

$$\begin{aligned}&\sum_{j\in\mathcal{J}}\sum_{t=r+1}^{r+H} t y^s_{i_1jt} \ge \sum_{j\in\mathcal{J}}\sum_{t=r+1}^{r+H} t x^1_{i_1jt} + \underline{a}^f \\ &-M\left(1-k^s_{i_1}\right),\ \forall i_1 \in \mathcal{I}_1, s \in \mathcal{S},\end{aligned} \tag{9}$$

$$\begin{aligned}&\sum_{j\in\mathcal{J}}\sum_{t=r+1}^{r+H} t y^s_{i_1jt} \le \sum_{j\in\mathcal{J}}\sum_{t=r+1}^{r+H} t x^1_{i_1jt} + \overline{a}^f \\ &+M\left(1-k^s_{i_1}\right),\ \forall i_1 \in \mathcal{I}_1, s \in \mathcal{S},\end{aligned} \tag{10}$$

$$\begin{aligned}&\sum_{j\in\mathcal{J}}\sum_{t=r+1}^{r+H} t z^s_{i_1jt} \ge \sum_{j\in\mathcal{J}}\sum_{t=r+1}^{r+H} t x^1_{i_1jt} + \underline{a}^p \\ &-Mk^s_{i_1},\ \forall i_1 \in \mathcal{I}_1, s \in \mathcal{S},\end{aligned} \tag{11}$$

$$\begin{aligned}&\sum_{j\in\mathcal{J}}\sum_{t=r+1}^{r+H} t z^s_{i_1jt} \le \sum_{j\in\mathcal{J}}\sum_{t=r+1}^{r+H} t x^1_{i_1jt} + \overline{a}^p \\ &+Mk^s_{i_1},\ \forall i_1 \in \mathcal{I}_1, s \in \mathcal{S}.\end{aligned} \tag{12}$$

where M represents a large number.

(d) Delivery time constraints: The completion time of the first repair of the returned products cannot be longer than the product's due date in order to guarantee on-time delivery, that is,

$$\sum_{j\in\mathcal{J}}\sum_{t=r+1}^{r+H} t x^1_{i_1jt} \le M_1,\ \forall i_1 \in \mathcal{I}_1, \tag{13}$$

$$\sum_{j\in\mathcal{J}}\sum_{t=r+1}^{r+H} t x^2_{i_2jt} \le M_2,\ \forall i_2 \in \mathcal{I}_2. \tag{14}$$

(e) Overtime and idle time calculation: Each workstation's repair overtime and idle capacity for processing $Q_i,\ i=1,\ 2,\ 3$ returns can be constrained and calculated as follows,

$$\begin{aligned}&o^{1s}_{jt} \ge \sum_{i_1\in\mathcal{I}_1} d^1_j x^1_{i_1jt} + \sum_{i_2\in\mathcal{I}_2} d^1_j x^2_{i_2jt} - c^1_{jt}\alpha_{jt}, \\ &\forall j \in \mathcal{J}, t \in \mathcal{T}, s \in \mathcal{S},\end{aligned} \tag{15}$$

$$\begin{aligned}&v^{1s}_{jt} \ge c^1_{jt}\alpha_{jt} - \sum_{i_1\in\mathcal{I}_1} d^1_j x^1_{i_1jt} - \sum_{i_2\in\mathcal{I}_2} d^1_j x^2_{i_2jt}, \\ &\forall j \in \mathcal{J},\ \ t \in \mathcal{T}, s \in \mathcal{S},\end{aligned} \tag{16}$$

$$o^{2s}_{jt} \ge \sum_{i_1\in\mathcal{I}_1} d^2_j y^s_{i_1jt} - c^2_{jt}\beta_{jt},\ \forall j \in \mathcal{J},\ t \in \mathcal{T}, s \in \mathcal{S}, \tag{17}$$

$$v^{2s}_{jt} \ge c^2_{jt}\beta_{jt} - \sum_{i_1\in\mathcal{I}_1} d^2_j y^s_{i_1jt},\ \forall j \in \mathcal{J},\ t \in \mathcal{T}, s \in \mathcal{S}, \tag{18}$$

$$o^{3s}_{jt} \ge \sum_{i_1\in\mathcal{I}_1} d^3_j z^s_{i_1jt} - c^3_{jt}\gamma_{jt},\ \forall j \in \mathcal{J},\ t \in \mathcal{T}, s \in \mathcal{S}, \tag{19}$$

$$v^{3s}_{jt} \ge c^3_{jt}\gamma_{jt} - \sum_{i_1\in\mathcal{I}_1} d^3_j z^s_{i_1jt},\ \forall j \in \mathcal{J},\ t \in \mathcal{T}, s \in \mathcal{S}. \tag{20}$$

(f) Decision variables ranges definition: The following constraints define the integrality and non-negativity restrictions on the decision variables.

$$\begin{aligned}&x^1_{i_1jt}, x^2_{i_2jt},\ y^s_{i_1jt},\ z^s_{i_1jt} \in \{0,1\},\ \ o^{is}_{jt},\ v^{is}_{jt} \ge 0, \\ &\forall i_1 \in \mathcal{I}_1, i_2 \in \mathcal{I}_2, j \in \mathcal{J},\ i=1,\ 2,\ 3, \\ &t = r+1, \ldots, r+H, s \in \mathcal{S}.\end{aligned} \tag{21}$$

(g) Objective function: The total process capacity overtime and idle cost of first and second repair operations for $Q_i,\ i=1,\ 2,\ 3$ returns for all workstations and all periods in the planning horizon at decision period r in scenario s is expressed as follows:

$$TC^s_{ov} = \sum_{j\in\mathcal{J}}\sum_{t\in\mathcal{T}}\sum_{i=1}^{3} c^i_b(o^{is}_{jt} + v^{is}_{jt}) \tag{22}$$

The sum of setup cost of repair operations for $Q_i,\ i=1,\ 2,\ 3$ returns for all workstations and all periods is expressed as follows:

$$TC_u = \sum_{j\in\mathcal{J}}\sum_{t\in\mathcal{T}} c^1_u\alpha_{jt} + c^2_u\beta_{jt} + c^3_u\gamma_{jt} \tag{23}$$

We model soft constraints to ensure the second repair of one return is processed at the same workstation as in its first repair. A penalty term is added to the objective function if this condition is not met. The total penalty cost in scenario s for all returns is expressed as follows:

$$TC_p^s = \sum_{i_1 \in \mathcal{I}_1} \sum_{j \in \mathcal{J}} c_p^f k_{i_1}^s \ (\sum_{t \in \mathcal{T}} y_{i_1 jt}^s - \sum_{t \in \mathcal{T}} x_{i_1 jt}^1)^2$$
$$+ c_p^p (1 - k_{i_1}^s)(\sum_{t \in \mathcal{T}} z_{i_1 jt}^s - \sum_{t \in \mathcal{T}} x_{i_1 jt}^1)^2 \quad (24)$$

The objective function is to minimize the total expected cost encompassing repair setup cost, capacity overtime and idle cost, and penalty cost as presented below:

$$min \ \sum_{s=1}^{S} p_s (TC_{ov}^s + TC_u + TC_p^s) \quad (25)$$

IV. Numerical Experiments

The above approach is conducted by using Gurobi 10.0.3 on a personal computer with a 2.40 GHz Intel Core i7-13700H and 16 GB of memory. Gurobi parameters use default settings, except MIPFocus is set to 3. First, we present basic data settings and design numerical experiments. Then, we conduct experiments to compare the performance of the proposed stochastic programming model and the deterministic model. Finally, we provide practical implications for better use of the proposed stochastic programming model and solution.

A. Data Settings

We introduce our basic data settings in this section. We use "pu" to denote the process capacity unit and "mu" to denote the monetary unit. For the period-related parameters, we set $R = 5$, $H = 8$, $M^1 = 3$, $M^2 = 6$, $\underline{a}^f = 2$, $\overline{a}^f = 3$, $\underline{a}^p = 4$, and $\overline{a}^p = 5$, respectively. Each period receives orders including 20 units of first returns that will require a second repair ($\mathcal{I}_1$) and 10 units of first returns that will not require a second repair ($\mathcal{I}_2$). The number of workstations is set to 3 and each workstation is well equipped to be capable of repairing $Q_i, \ i = 1, \ 2, \ 3$ returns, *i.e.*, $\alpha_{jt} = \beta_{jt} = \gamma_{jt} = 1, \forall \ j \in \mathcal{J}, \ t \in \mathcal{T}$. Each workstation has a regular total process capacity level $c_{jt} = 680 \ pu$. Since some capacity is occupied by pre-arranged products before planning, each workstation has no available capacity for Q_2 returns during the first two periods and no available capacity for Q_3 returns during the first four periods. Each workstation has a capacity of 150 pu for Q_2 returns and 280 pu for Q_3 returns in the subsequent periods. In terms of repair capacity for Q_1 returns, we set $c_{jt}^1 = 250 \ pu, \ \forall \ j \in \mathcal{J}, \quad t = r+1, \ldots, r+H$. We set $d_j^1 = 60 \ pu, d_j^2 = 90 \ pu$, and $d_j^3 = 150 \ pu$. For the cost parameters, $c_u^1 = c_b^1 = 1 \ mu, c_u^2 = c_b^2 = 2 \ mu$, and $c_u^3 = c_b^3 = 3 \ mu$ respectively. c_p^f and c_p^p take value as 1 mu and 1.5 mu, respectively.

B. The Value of the Stochastic Solution

Based on the above basic parameter settings, we investigate three cases with three levels of the probability that the quality level of the second return is Q_2, *i.e.*, p_{i_1}= 0.4, 0.5 and 0.6. For each case, eight instances are included, and six scenarios are randomly generated in each instance. Table I shows the comparison results of objective value for different solutions. The objective values of solutions obtained by the proposed stochastic programming model are summarized in the "SP-SAA" row. The "DM" row summarizes the average objective values from the existing deterministic model, with their associated standard deviation in the "Stdev" row. Keeping the remaining parameters unchanged, the deterministic model is solved for each scenario in each instance. Then, we use the proposed stochastic programming model to solve each instance. In all 24 instances across 3 cases, the objective values of SP-SAA are lower than the average objective values from DM. Assuming that the data are normal, the limits of the intervals are computed using quantiles of a normal distribution with a 95% confidence interval. Every interval result in this article is obtained using the same methodology [14]. The solution provided by stochastic programming can bear an average cost of 17017 mu, 15018 mu, and 14771 mu for Case 1, Case 2, and Case 3 respectively. It is 6.6%, 9.4% and 9.8% lower than the average cost of the solution yielded by deterministic model for 3 cases, respectively. The p-values of the two-sample t-test for the stochastic solutions and deterministic solutions across three cases are 0.036, 0.0005, and 0.011, all less than the significance level of 0.05. The aforementioned results demonstrate that the stochastic solutions perform statistically significantly better than the average performance of solutions obtained by the deterministic model under different scenarios.

C. Practical Implications

We offer industrial practitioners guidelines for implementing stochastic programming solutions. Practitioners implement consistent first-stage repair decisions, then receive and assess second returns to align actual quality with pre-considered scenarios, implementing corresponding second repair decisions. Alternatively, the model can be re-optimized based on actual return quality. The objective value of stochastic programming represents expected performance, but its achievability is uncertain.

Some managerial insights are also provided by discussing the solutions for the aforementioned 3 cases, where the demand is redesigned to vary with the time period but maintains an average level of $I_1 = 20$ and $I_2 = 10$. Fig. 2 illustrates the results of capacity allocation in the three cases. For each case, the optimal capacity allocation generally exhibits the inverted parabola pattern, where the capacity first increases, then decreases. And the peak of Case 1 appears earlier than the peaks of Cases 2 and 3. The capacity peak allocated to Q_1 returns appears the earliest, followed by Q_2 returns, with Q_3 returns exhibiting the latest. The difference is that in the three cases, as the number of Q_2 returns increases, more capacity is allocated to Q_2, and less capacity is allocated to Q_3 in the optimal solution with a lower total cost. Practitioners can typically allocate capacity based on the above insights, even without solving the problem.

TABLE I
THE COMPARISON RESULTS OF SOLUTION PERFORMANCE

Case	p_{i_1}	Method	I1	I2	I3	I4	I5	I6	I7	I8	Average	95% CI
		DM	18159	18864	16521	19070	17832	18002	18421	18910	18222	[16609, 19836]
1	0.4	Stdev	2109	1190	1521	2062	1445	1476	1882	1865		
		SP-SAA	17356	18100	15302	18176	15495	17032	18335	16340	17017	[14665, 19369]
		DM	16357	15893	15924	16702	17232	18037	16278	16232	16582	[15152, 18012]
2	0.5	Stdev	954	1056	1624	1374	1893	2340	1225	1165		
		SP-SAA	13748	14835	15134	15917	14911	15424	14554	15624	15018	[13688, 16349]
		DM	14829	15163	15822	17886	17725	16067	15951	16263	16213	[14073, 18354]
3	0.6	Stdev	541	783	1295	4408	2611	2066	1409	2471		
		SP-SAA	14390	14953	14218	16785	14009	14673	14486	14651	14771	[13077, 16464]

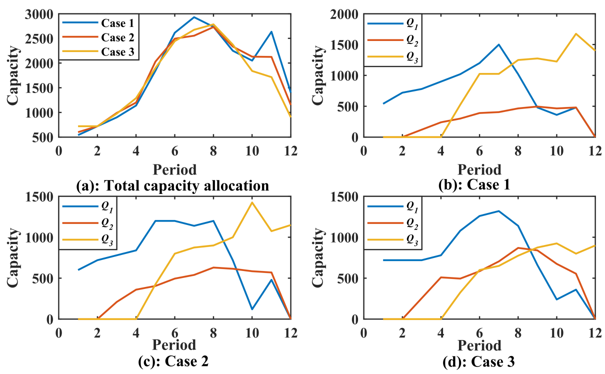

Fig. 2. The results of capacity allocation in the three cases.

V. CONCLUSION

This work investigates a make-to-order circular production system with multiple parallel workstations, ensuring that first and second repairs for returns occur at the same workstation. The aim is to minimize overtime and idle capacity while maintaining processing traceability. The production planning and allocation problem is formulated as a SAA based stochastic mixed-integer programming model to address quality uncertainty in second returns.Two stage decision variables are introduced to determine the allocation of two repairs for a return to a workstation at a period. Rolling plans are generated using a sequential decision-making framework to determine the production plan over a finite horizon. Numerical results illustrate the value of the solutions obtained by proposed stochastic model compared with the deterministic solutions. Problem-oriented management insights are also explored by investigating various cases to guide practical applications. To fairly compare the performance of the proposed stochastic solutions with the deterministic solutions, in this paper, the cases studied can be considered as small-scale problems such that the exact solution of the proposed SAA based stochastic programming model is obtained using Gurobi. The numerical cases can be easily extended to medium and large problem scales for practical applications but only feasible solutions can be provided. In the future work, efficient exact or heuristic algorithms will be developed to solve large scale problem. Production planning and allocation problems for multiple orders, multiple times returns, and different objectives can also be studied. In addition, we will develop robust optimization or distributionally robust optimization model for the problem to address the limitations in stochastic programming model.

REFERENCES

[1] D. Reike, W. J. Vermeulen, and S. Witjes, "The circular economy: new or refurbished as CE 3.0?—exploring controversies in the conceptualization of the circular economy through a focus on history and resource value retention options," *Resour. Conserv. Recycl.*, vol. 135, pp. 246–264, Nov. 2018.

[2] X. Schepler, N. Absi, and A. Jeanjean, "Refurbishment and remanufacturing planning model for pre-owned consumer electronics," *Int. J. Prod. Res.*, pp. 1–23, Jun. 2023.

[3] F. Quezada, C. Gicquel, and S. Kedad-Sidhoum, "A stochastic dual dynamic integer programming based approach for remanufacturing planning under uncertainty," *Int. J. Prod. Res.*, vol. 61, no. 17, pp. 5992–6012, Sep. 2023.

[4] H. S. Heese, K. Cattani, G. Ferrer, W. Gilland, and A. V. Roth, "Competitive advantage through take-back of used products," *Eur. J. Oper. Res.*, vol. 164, no. 1, pp. 143–157, Jan. 2005.

[5] N. Ndhaief, R. Nidhal, A. Hajji, and O. Bistorin, "Environmental issue in an integrated production and maintenance control of unreliable manufacturing/remanufacturing systems," *Int. J. Prod. Res.*, vol. 58, no. 14, pp. 4182–4200, Aug. 2020.

[6] W. Zhang, Y. Zheng, and R. Ahmad, "An energy-efficient multi-objective integrated process planning and scheduling for a flexible job-shop-type remanufacturing system," *Adv. Eng. Inform.*, vol. 56, May. 2023, Art. no. 102010.

[7] Q. Jin, S. Zhang, J. Xu, K. Jiang, Y. Cheng, and W. Zhang, "Remanufacturing system scheduling of batch products with the consideration of dynamic changes in machine efficiency using an improved artificial bee colony algorithm," *Comput. Ind. Eng.*, vol. 187, Dec. 2023, Art. no. 109817.

[8] İ. Yanıkoğlu, and M. Denizel, "The value of quality grading in remanufacturing under quality level uncertainty," *Int. J. Prod. Res.*, vol. 59, no. 3, pp. 839–859, Jan. 2021.

[9] K. N. Reddy, and A. Kumar, "Capacity investment and inventory planning for a hybrid manufacturing–remanufacturing system in the circular economy," *Int. J. Prod. Res.*, vol. 59, no. 8, pp. 2450–2478, Mar. 2021.

[10] X. Chang, H. Xia, H. Zhu, T. Fan, and H. Zhao, "Production decisions in a hybrid manufacturing–remanufacturing system with carbon cap and trade mechanism," *Int. J. Prod. Econ.*, vol. 162, pp. 160–173, Feb. 2015.

[11] X. Shen, S. C. Du, Y. N. Sun, P. Z. Sun, R. Law, and E. Q. Wu, "Advance scheduling for chronic care under online or offline revisit uncertainty," *IEEE Trans. Autom. Sci. Eng.*, Aug. 2023.

[12] M. Poursoltani, E. Delage, and A. Georghiou, "Robust data-driven prescriptiveness optimization," 2023, *arXiv preprint arXiv:2306.05937*.

[13] P. Guo, Z. Chen, Y. Yang, and R. Miao, "A multistage simulation-optimization-integrated methodology framework for user-oriented electric vehicle carsharing reallocation under dynamic price subsidy," *Energy*, vol. 290, Dec. 2023, Art. no. 130207.

[14] M. Zhang, E. Pastore, A. Alfieri, and A. Matta, "Buffer allocation problem in production flow lines: A new Benders-decomposition-based exact solution approach," *IISE Trans.*, vol. 54, no. 5, pp. 421–434, Apr. 2022.